

The Mathematics of Hope:

Moving from Performance to Learning in Mathematics Classrooms

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In 2006 a trade book appeared on bookshelves that would ultimately have one of the biggest impacts of any research volume ever published in education. In *Mindset: The New Psychology of Success* Carol Dweck summarized key findings from her research on the nature and impact of mindsets. The book quickly became a *New York Times* best-seller and was translated into more than twenty languages. Dweck's decades of research with subjects of various ages showed that students with a "growth mindset"—who believe that intelligence and "smartness" can be learned—go on to higher levels of achievement, engagement, and persistence. The implications of this mindset are profound, especially for students of mathematics.

Mathematics, more than any other subject, has the power to crush students' confidence (Boaler 2009). The reasons are related both to the teaching methods that prevail in U.S. math classrooms and the fixed ideas about mathematics held by the majority of the U.S. population and passed on to our children from birth. One of the most damaging mathematics myths propagated in classrooms and homes is that math is a gift, that some people are naturally good at math and some are not (Boaler 2013a, 2013b). This idea is strangely cherished in the Western world but virtually absent in Eastern countries such as China and Japan that top the world in mathematics achievement (PISA 2012).

New scientific evidence showing the incredible capacity of the brain to change, rewire, and grow in a really short time (Maguire et al. 2006) suggests that all students can learn mathematics to high levels with good teaching experiences. Traditional educators believe that some students do not have the brains to be able to work on complex mathematics, but it is working on complex mathematics that enables brain connections to develop. Students can grasp high-level ideas but they will not develop the brain connections that allow them to do so if they are given low-level work and negative messages about their own potential (Boaler & Foster 2014).

As I work with schools and districts, encouraging mathematics teaching that promotes growth rather than fixed mindsets (www.youcubed.org), a critical requirement is that teachers offer mathematics as a learning subject, not a performance subject. Most students asked what they think their role is in math classrooms say it is to answer questions correctly. They don't think they are in math classrooms to appreciate the beauty of mathematics, to explore the rich set of connections that make up the subject, or even to learn about the applicability of the subject; they think they are in math classrooms to perform. This was brought home to me recently when a colleague, Rachel Lambert, told me her six-year old son had come home saying he didn't like math; when she asked him why, he said that "math is too much answer time and not enough learning time." Students

"Math is too much answer time and not enough learning time"
-6 year old

from kindergarten upward realize that math is different from other subjects: learning gives way to answering questions and taking tests - performing.

For students to see mathematics as a subject of learning, not performing, they need tasks and questions in math class that have space to learn built in. When students spend all their time in math class answering discrete questions to which the answers are either right or wrong, it is very difficult to develop a growth mindset or to believe that mathematics is about growth and learning. When teachers ask students to find the area of a rectangle with sides of 12 feet and 2 feet, for example, students are being asked to perform a single calculation correctly. When teachers ask students to find a rectangle with an area of 24 square feet, students are being asked to use their imagination, to think; they need to consider various rectangles and think about the relationship between length and width. Students can propose different rectangles and discuss the equivalence of area in different shapes. In the first example students are answering a question on area; in the second they are learning about rectangles, dimensions, and area. If a mathematics question or task does not have space within it to think, learn, and discuss, its potential as a learning task is limited. Tasks that are particularly valuable are those that have a low floor and a high ceiling - that is, anyone can access them, but they can be taken to very high levels. (For examples of high-quality math tasks and low-floor, high-ceiling tasks, see www.youcubed.org, watch <https://www.youtube.com/watch?v=pOOW0hQgVPQ>, and see <http://nrich.maths.org>.)

“For students to see mathematics as a subject of learning, not performing, they need tasks and questions in math class that have space to learn built in.”

Mathematics and Mistakes

Research has recently shown something stunning—when students make a mistake in math, their brain grows, synapses fire, and connections are made; when they do the work correctly, there is no brain growth (Moser et al. 2011). This finding suggests that we want students to make mistakes in math class and that students should not view mistakes as learning failures but as learning achievements (Boaler 2013a). Students do not, as many assume, need to revisit a mistake and correct it to experience brain growth, although that is always helpful; brain growth comes from the experience of struggle. When students struggle with mathematics, their brains grow; being outside their comfort zone is an extremely important place to be.

Mathematics classrooms throughout the U.S. are often set up to make students feel good by giving them lots of questions they can answer. Teachers believe that mistakes and struggle are unproductive and try to shelter students from them. This culture needs to change. While I was sitting in an elementary classroom in Shanghai recently, the principal leaned over to tell me that the teacher was calling on students who had made mistakes to share with the whole class so that they could all learn. The students seemed pleased to be given the opportunity to share their incorrect thinking. Instead of classrooms filled with short questions students are intended to get right or wrong, mathematics classrooms need to be filled with open-ended tasks that include space for learning as well as space for struggle and growth (www.youcubed.org).

For mathematics to become a learning subject with room for mistakes and growth, teachers need to make students feel good about mistakes and comfortable with struggle. When I taught a recent online

class and shared the mistakes research with forward-thinking mathematics teachers, they came up with a range of ways for getting students to value mistakes (<http://www.telegraph.co.uk/education/universityeducation/10414989/University-education-maturing-of-the-Mooc.html>, <http://tinyurl.com/oz4u4ga>). One teacher, on the first day of school, asked her students to crumple up a piece of paper and throw it at the wall in a way that expressed the feelings they had when they made a mistake in math. She then asked them to retrieve the paper, uncrumple, and use a colored marker to highlight all the creases left on the paper; these, she explained, symbolized the brain growth that comes from mistakes. She had the students hang on to their brain growth diagrams for the school year.

Other changes need to happen as well. Mathematics teachers need to stop frequent, timed testing; replace grades with diagnostic feedback (Black et al. 2002; Boaler & Foster 2014); and deemphasize speed, so that the students who think slowly and deeply are not led to believe they are not capable (Boaler 2014). Perhaps most significantly and most radically, schools should also remove fixed student groupings that transmit fixed mindset messages and replace them with flexible groupings that recognize that students have different strengths at different times (Boaler 2009; Boaler & Foster 2014).

Stop	Replace with
Frequent timed tests	Number Talks
Grades	Diagnostic feedback
Emphasizing Speed	Time to think slowly and deeply
Ability Grouping	Heterogeneous and flexible groupings

Fortunately these changes are entirely consistent with what is known about good teaching and learning. Decades of research show that when students engage actively with mathematics—work on long, applied problems with room for struggle and growth—and receive positive messages about their potential, they succeed (Boaler 2009; Schoenfeld 2002). The new Common Core mathematics standards (<http://www.corestandards.org/>) include a set of eight mathematical practices that require students to work in these ways, and they are a step in the right direction.

Currently three fifths of U.S. students fail mathematics, and mathematics is a harshly inequitable subject (Kozol 2012; Silva & White 2013). When our classrooms change—when students are encouraged to believe they can be successful in mathematics and are taught using the high-quality teaching methods they deserve—the landscape of mathematics teaching and learning in the United States will change forever (Boaler & Foster 2014). We will have many more confident and capable mathematics learners, and they will go on to become teachers of mathematics who inspire future generations to further success in science, technology, and mathematics.

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